



## Shape description of MR brain images

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### Abstract

The paper provides a classification of shape representation and description techniques and a review of some of the most important techniques. We examine shape description techniques on the reconstructed boundaries of brain structures (white matter) in the magnetic-resonance (MR) images using the Fourier descriptors for various frequency components in order to characterize brain structures.

### 1. Introduction

An object consists of interior points or contents surrounded by a boundary. Objects can be characterized by certain features: grey levels, textures, edges, boundaries, shapes, locations, etc. The shape is the most important visual feature of an object generally considered as the form of the object boundary.

A shape boundary of the object, which is often called the object contour, considered as a closed, planar curve that can be defined as the function in: the explicit form as  $y = y(x)$ ; the implicit form, for example  $f(x, y) = 0$ ; the parametric form by natural parameterization  $c(l) = (x(l), y(l))$ , every point  $(x_0, y_0) \dots (x_{N-1}, y_{N-1})$  of the contour  $c$  of the object can be represented by taking the arc length  $l$  as a parameter with  $0 \leq l \leq L$ , where  $L$  is the length of the contour; the parametric form in the polar coordinates as  $\tau(l) = (d(l), \theta(l))$ ; the parametric form in the complex plane,  $z(l) = x(l) + i \cdot y(l)$ .

In order to describe and classify brain structures there is used the shape analysis based on various shape representation and description techniques for precise quantification and measurement.

Shape representation techniques that operate on the segmented image, represent the object in a suitable form and then as a post-processing technique uses a shape description which generates descriptors of the shape. Descriptors of

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the shape are a set of numbers describing specific characteristics of an object and considered as shape parameters of these objects they can be compared and recognized by matching. Descriptors should not depend on geometrical transformations such as translation, rotation and changes of scale. It means that they should be invariant to these three transformations and do not change the shape of an object.

## **2. Classification of shape representation and description techniques**

Generally there is no accepted methodology of shape representation and description techniques [1], a good review on techniques can be found in [1-5], but common shape representation and description techniques can be classified into two classes [3-6]:

- contour-based methods, also called external methods, based on information extracted from the object boundary and its features (in terms of boundary length);
- region-based methods, also called internal methods, based on the information extracted from the shape region occupied by the object.

Each of these classes, contour-based and region-based methods can be divided into two types of approaches based on how information is extracted [3]:

- structural approach, also called discrete, decompose the shape boundary or shape region into segments or sub-parts, called primitives, using some criterion;
- global approach, also called continuous, does not divide shape boundary or whole shape region into segments.

The hierarchy of the classification of shape representation and description techniques is illustrated in Fig. 1.

Shape representation and description techniques can also be classified into three categories: local, global and medial (combination of local and global properties) techniques [2]. Local and global shape representation techniques differ on the basis of the access of the shape properties: local provides access of the shape boundary points, allows to extract specific, localized characteristics (chain code, signature, polygon, convex hull, spline); global techniques representing the overall shape is suitable for extracting more general shape characteristics (Fourier descriptors, moments, Hough transform). Medial shape representation is combination of local and global properties (media axis, skeleton). Local description techniques or local descriptors mainly based on differential geometry of the shape, allow extract local structural shape characteristics such as the boundary curvature or corners. On to the contrary, global shape descriptors provide global information based on the overall shape contour properties such characteristics as perimeter, area, compactness, bending

energy, eccentricity, Fourier descriptors, fractal dimension. From the medial shape representation there can be generated medial descriptors local: normal (end, branch) points, shape width; global: medial orientation, diameter.

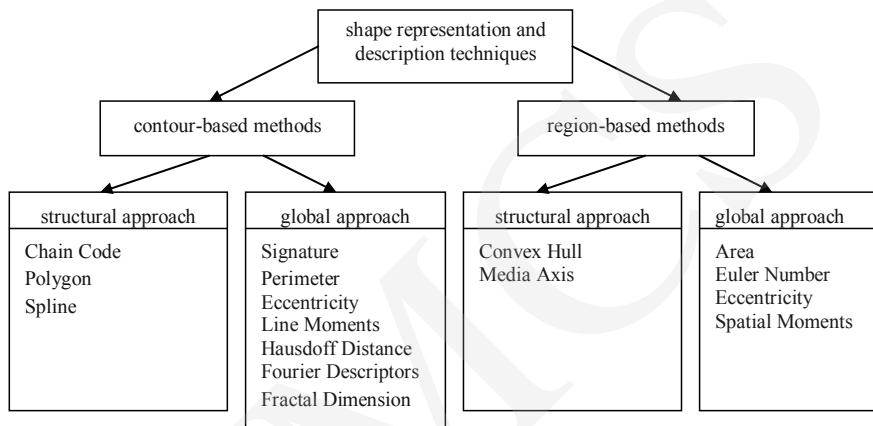


Fig. 1. Classification of shape representation and description techniques

### 3. Shape representation techniques

In 1961 Freeman [7] devised a technique of boundary coding, called chain coding. Chain code and other structural contour-based shape representation methods are decomposed object boundaries into segments, primitives differing in selection and organization. In the general form they can be defined as:  $K = k_1, k_2, \dots, k_n$ , where  $k_i$  can represent the elements like a chain code, a side of a polygon, a spline, and others.

The chain code is one of the simplest representation techniques in which shape of an object is represented by quantifying the position of points on its boundary. It achieves representation by analyzing each point on the boundary, run clockwise, and used to obtain a sequentially connected list of boundary or contour points based on 4- or 8-connectivity direction vectors.

In the case of chain code representation an object can be completely reconstructed. The disadvantage of the chain code: for complex objects it may be too large: is not invariant to translation: chain code changes with the selected starting point, should be the same starting point for comparing chain codes the different objects; small variations in the contour for example due to noise, arbitrary distortion or a chosen segmentation method gives a different code.

A polygonal representation [8,9] approximates an object contour by a polygon subdividing the contour points into groups, each of which is to be approximated by a polygonal side or a linear segment. The most accurate approximation can be defined as the number of linear segments of the polygon

being equal to the number of points of the object contour. The most known methods of polygonal representation are based on the use of some approximation criteria such as the minimal error, the minimal polygon perimeter, the maximal internal polygon area, the minimal external polygon area.

Splines have been used to describe curves that can be divided into intervals. A spline of the order  $d$  is a piecewise polynomial function, consisting of concatenated polynomial segments or intervals, or spans, each of some polynomial order  $d$ , joined together at breakpoints or control points are called knots. Usually the polynomial order  $d$  is fixed at quadratic ( $d=3$ , the order of a polynomial is the number of its coefficients), or cubic ( $d=4$ ). The boundary of an object can be described by the set of curve intervals  $v(l) = \{v_1(l), \dots, v_n(l)\}$ , where the parameter  $l$  is arc length, as given by [10]:

$$v(l) = v_1(l) + \sum_{i=1}^{n-1} q_i (l - l_i)^m, \quad (1)$$

where  $m$  is degree  $m = d - 1$ ,  $(l - l_i)^m$  is zero then  $(l - l_i) \leq 0$  and  $q_i$  is a constant proportional to the  $m$  derivative of  $v(l)$  with respect to  $l$  at  $l = l_i$ .

Splines have a good property of minimizing curvature though they approximate a given function with a curve having the minimum average curvature. It provides compact “natural” representation of curves and allows for complete reconstruction of an object.

A signature is a one-dimensional function representation of an object boundary, which can be constructed in various ways from the given two-dimensional function of an object boundary. Chain coding, radius (angle), tangent angle, chord-length representations are all signatures and their many other forms have been proposed in literature [1,3,11].

For example, the distance-versus-angle signature (the radius-angle) representation of the shape can be obtained as a sequence of the distances from the interior point usually centroid to each boundary point as a radius  $r(\theta)$  function of the angle  $\theta$ .  $\theta$  is the angle between the radius, drawn from the interior point to each contour element, and the parallel x- or y- axes where  $0 \leq \theta \leq 2\pi$ . This representation allows to reconstruct completely the object but it can be applied only for star-shaped objects, when  $r(\theta)$  is single-valued. The radius-angle representation is not suitable for non star-shaped objects.

A Fourier representation [5,12] decomposes a shape contour, into a set of numbers that represent its frequency components or Fourier descriptors, obtained by its Fourier transform. Each Fourier descriptor describes a global property of the shape of an object.

Each point on the contour given parametrically can be represented by a complex number, the real and imaginary parts of which are the  $x$  and  $y$  coordinates of the points  $(x_0, y_0) \dots (x_{N-1}, y_{N-1})$  in the form  $f_s(l) = x(l)$  real and

$f_y(l) = y(l)$  imaginary parts. This allows the contour to be expressed as complex periodic function as:  $f(l) = f_x(l) + j \cdot f_y(l)$ ,  $j = \sqrt{-1}$ .

The discrete Fourier transform of  $f(l)$  is [7,13]:

$$T(u) = \frac{1}{N} \sum_{l=0}^{N-1} f(l) e^{-2j \cdot \pi \cdot u \cdot l / N} \quad (2)$$

for  $u = 0 \dots N-1$ .  $1/N$  multiplier in front of the Fourier transform sometimes is placed in front of the inverse transform [12] (as in our application). Fourier descriptors of the contour are given by the complex coefficients  $T(u)$ . The contour function  $f(l)$  can be recovered with the inverse transform of  $T(u)$ :

$$f(l) = \sum_{u=0}^{N-1} T(u) e^{2j \cdot \pi \cdot u \cdot l / N} \quad (3)$$

for  $l = 0 \dots N-1$ . However, instead of all the  $T(u)$  coefficients, only the first  $M$  coefficients are used to recover the contour, then the contour is approximated by

$$\hat{f}(l) = \sum_{u=0}^{M-1} T(u) e^{2j \cdot \pi \cdot u \cdot l / N} \quad (4)$$

for  $l = 0 \dots N-1$ , although only  $M$  terms are used to obtain each component of  $\hat{f}(l)$ . The same number of points still exists in the approximate contour, but fewer Fourier coefficients are used for reconstruction.

The infinite number of moments provides an effective description of the shape contour in terms of statistical analysis. Moment descriptors are very useful and practical being concentrated on statistical properties of the closed contour of an object by analyzing the distribution (probability density function) of values in a discrete binary image function – called spatial moments [7] or region-based moment, or by analyzing the distribution of values in a function of an object closed contour – called line moments [12] or contour-based moments.

In order to obtain the contour-based moments along the contour assuming that the contour of an object is defined parametrically by  $c(l) = (x(l), y(l))$ . The contour-based moments are given by

$$m_{pq}^c = \sum_{i=0}^{N-1} (x(l_i))^p (y(l_i))^q \quad (5)$$

The central contour-based moments can be obtained by

$$\mu_{pq}^c = \sum_{i=0}^{N-1} (x(l_i) - \bar{x}(l_i))^p (y(l_i) - \bar{y}(l_i))^q, \quad (6)$$

where  $\bar{x}(l_i) = m_{10}^c / m_{00}^c$  and  $\bar{y}(l_i) = m_{01}^c / m_{00}^c$ ,

$m_{00}^c$  corresponds to the length of the object boundary.

The scale invariant central contour-based moments will be defined as

$$\eta_{pq}^c = \mu_{pq}^c \mu_{00}^{c-\beta}, \text{ where } \beta = p + q + 1. \quad (7)$$

A set of invariant moments with respect to translation, rotation, and scaling can be obtained from the second- and third-order moments. Some examples are:

$$\begin{aligned}\phi_1 &= \eta_{20}^c + \eta_{02}^c & \phi_2 &= (\eta_{20}^c - \eta_{02}^c)^2 + 4\eta_{11}^{c^2} \\ \phi_3 &= (\eta_{30}^c - 3\eta_{12}^c)^2 + (3\eta_{21}^c - \eta_{03}^c)^2 & \phi_4 &= (\eta_{30}^c + \eta_{12}^c)^2 + (\eta_{21}^c + \eta_{03}^c)^2\end{aligned}\quad (8)$$

Some shape descriptors which can contribute to classification of objects are based on moments. The orientation of an object, defined as the direction along which the object is most elongated is computed as the angle  $\theta$  between the  $x$  axis and the axis around which the object can be rotated

$$\theta = \frac{1}{2} \operatorname{atan} \frac{2\mu_{11}^c}{\mu_{20}^c - \mu_{02}^c}. \quad (9)$$

The eccentricity  $\varepsilon$  of an object ranges from 0 for a circular object to 1 for a straight line and can be measured as

$$\varepsilon = \frac{(\mu_{20}^c - \mu_{02}^c)^2 + 4\mu_{11}^{c^2}}{(\mu_{20}^c + \mu_{02}^c)^2}. \quad (10)$$

#### 4. Shape description techniques

An important local shape descriptor is curvature. The curvature  $k$  at the point  $p$  along a contour is defined as the rate of change of the slope angle  $\theta(l)$  in tangent line  $l(l)$  and positive  $x$ -axis. Assuming that the contour is defined in an explicit form ( $y = y(x)$ ) the slope angle in the tangent direction is given by [13]:

$$\theta(l) = \tan^{-1} \frac{dy}{dx}. \quad (11)$$

The curvature of any point on the contour is then given by the rate of change of slope  $\theta(l)$

$$k(l) = \frac{d\theta(l)}{dl}. \quad (12)$$

The curvature for the contour defined parametrically  $c(l) = (x(l), y(l))$  is given by

$$k(l) = \frac{\dot{x}(l)\ddot{y}(l) - \dot{y}(l)\ddot{x}(l)}{(\dot{x}(l)^2 + \dot{y}(l)^2)^{3/2}}. \quad (13)$$

This relationship is the standard measure of a parametric curve curvature and it is called the curvature function [5]. Curvature can be used to characterize contour points as convex if the curvature at point  $P$  is positive and concave if it is negative. Finding positive and negative local maxima of the curvature (local maxima of the absolute curvature) is used for corner detection.

Common global shape descriptors of an object  $X$  are perimeter, area, compactness. The perimeter can be estimated by adding the distance between successive points in a contour, as

$$P(X) = \sum_{i=0}^{N-1} \sqrt{(x(l_i) - x(l_{i-1}))^2 + (y(l_i) - y(l_{i-1}))^2}. \quad (14)$$

The area of an object can be obtained as

$$A(X) = \frac{1}{2} \left| \sum_{i=0}^{N-1} (x(l_i) \cdot y(l_{i+1})) - (y(l_i) \cdot x(l_{i+1})) \right|. \quad (15)$$

Compactness, determined by using the perimeter  $P(X)$  and the area  $A(X)$  of an object  $X$ , defined as

$$C(X) = P(X)^2 / A(X), \quad (16)$$

measures roundness of a shape that is how close an object is to the circle. The value of compactness is minimal for the circular shape ( $4\pi \approx 12.57$  for the circle) that encloses a given area with the shortest perimeter. The value increases with the increasing shape complexity. However, the perimeter and the area are invariant to translation and rotation and are not invariant to scaling the compactness. They are invariant to all geometrical transformations: translation, rotation and scaling. Thus compactness is useful to distinguish objects independently of their orientation and size on the image plane. Sometimes the normalized variant is used in the form  $C'(X) = 1 - 4\pi/C(X)$  ranging between zero and one.

Shape of an object can be represented by its bending energy, defined as the energy necessary to bend a rod to a desired shape and is obtained as the sum of squared contour curvature  $k(l)$  (being defined in Eq. 12, 13) over the shape contour having length  $L$ :

$$B(X) = \frac{1}{L} \sum_{i=0}^{N-1} k(l_i)^2. \quad (17)$$

The fractal dimension is often considered as a parameter describing morphological complexity of objects. The fractal dimension value reflects alterations of structural properties. One of the methods, that we used, to estimate fractal dimension is called the hand and dividers or yardstick method [13], based on perimeter estimation using step or “yardstick” of different sizes. This dependence can be mathematically expressed as the relation  $P(X) \propto \lambda^{1-FD}$ , where  $P(X)$  is the perimeter of the object  $X$ ,  $\lambda$  is a step size and  $FD$  is the fractal dimension. The fractal dimension  $FD$  of the object can be estimated from a linear regression defined by  $\log(P(X)) = (1 - FD) \cdot \log(\lambda) + const.$

### 5. Experiment

In order to describe and classify brain structures, shape description methods are employed. Fig.2 (a1, a2) shows two MR images, one of a patient with severe deformations of the white brain matter (Patient A) and one of a patient with only subtle deformations (Patient B). The object boundaries (shown in Fig. 2 a2, b2) of white brain matters are reconstructed using the Fourier descriptors for different levels of  $M$  (given in Eq. (4)). As illustrated in Fig.2 a3-a8, b3-b8, higher level of  $M$  accounts for fine detail, more complex shapes are reconstructed, when  $M$  becomes lower more detail is lost on the boundary of an object. The task is to evaluate the resulting reconstructed boundaries of different levels of  $M$  with shape descriptors in order to describe and classify brain structures and examine shape description techniques.

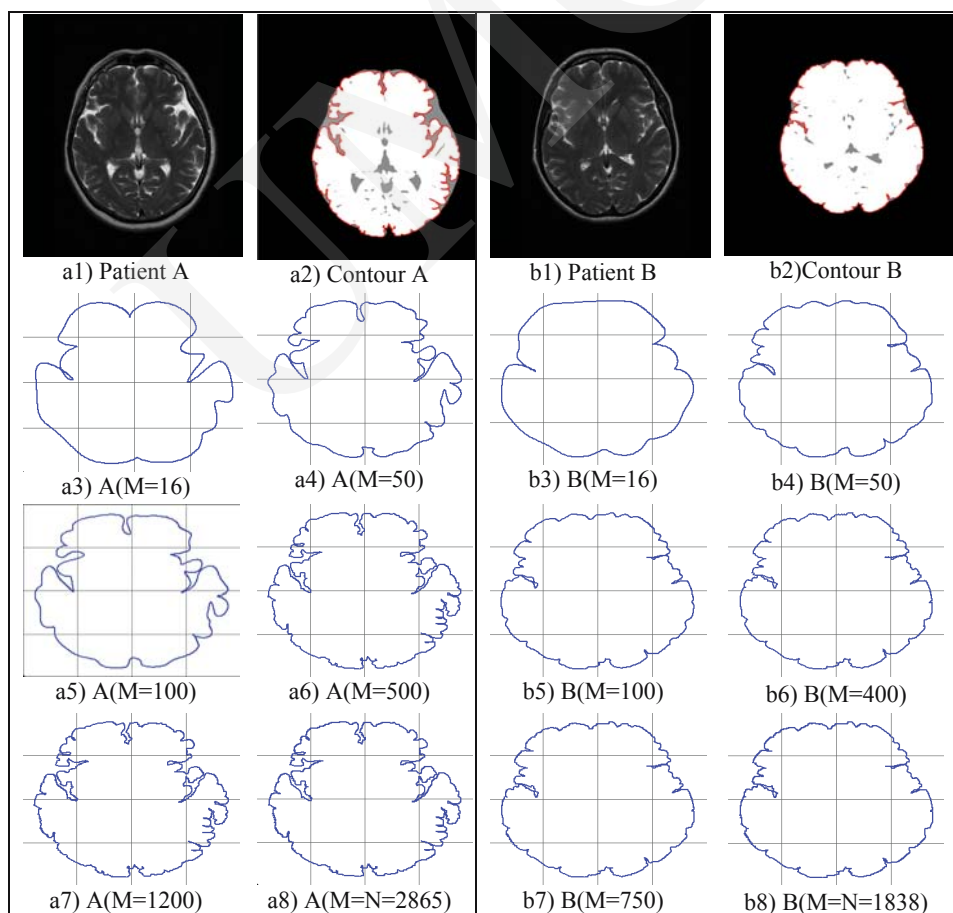


Fig. 2. Selected samples of reconstruction boundaries of white matters patient A a1, patient B b2, using the Fourier descriptors for various levels of  $M$  (frequency components)

Table 1. Values for the selected levels of  $M$  (frequency component, given in Eq. 4):  $A$  – area (pixels),  $P$  – perimeter (pixels),  $C$  – compactness,  $B$  – bending energy,  $\varepsilon$  – eccentricity,  $\phi_1$  – combination of scale invariant central moments (given in Eq. 8),  $FD$ - fractal dimension

Table 1. a) Patient A

No	$M$	$A$	$P$	$C$	$B$	$E$	$\phi_1$	$FD$
1	2800	68558	7240.51	764.68	4.22	0.35106	0.00435	0.39106
2	2500	67988	6919.88	704.31	4.47	0.32921	0.00498	0.33555
3	2300	68019	6725.54	665.00	4.64	0.31544	0.00543	0.33375
4	2000	68167	6472.01	614.47	4.77	0.29710	0.00610	0.32819
5	1800	68302	6355.66	591.40	4.78	0.28984	0.00645	0.32280
6	1500	68568	5444.62	432.32	5.39	0.21809	0.01025	0.31716
7	1200	68550	3202.59	149.62	7.23	0.01229	0.05046	0.29665
8	900	68656	2652.58	102.48	8.25	0.00592	0.08887	0.29349
9	500	68781	2582.21	96.94	8.34	0.01109	0.09635	0.29135
10	100	68711	2285.76	76.04	13.92	0.05833	0.13909	0.27266
11	50	68199	1993.28	58.26	15.36	0.17604	0.20966	0.23536
12	16	66698	1491.61	33.36	11.07	0.79279	0.50097	0.14445

Table 1. b) Patient B

No	$M$	$A$	$P$	$C$	$B$	$\varepsilon$	$\phi_1$	$FD$
1	1800	71955	4664.45	302.37	4.17	0.34362	0.00970	0.18780
2	1700	71454	4535.74	287.91	5.15	0.33205	0.01055	0.15253
3	1600	71463	4509.02	284.50	5.58	0.32821	0.01074	0.14442
4	1500	71567	4465.51	278.63	5.29	0.32407	0.01107	0.14108
5	1300	71656	4282.43	255.93	5.44	0.30487	0.01255	0.13982
6	1100	71804	3778.57	198.84	5.82	0.24756	0.01827	0.14062
7	900	71826	2469.27	84.89	7.21	0.06251	0.06558	0.13672
8	750	71852	1915.12	51.05	8.04	0.00167	0.14062	0.13664
9	400	71921	1581.18	34.76	8.93	0.02346	0.25016	0.13537
10	100	71961	1532.31	32.63	9.74	0.03667	0.27568	0.12530
11	50	71852	1439.29	28.83	12.31	0.07024	0.33208	0.10789
12	16	71303	1261.43	22.32	12.56	0.19079	0.49248	0.07113

## 6. Results

We have evaluated shapes, which reconstructed using the Fourier descriptors for different levels of  $M$ , for patients A and B with the suggested shape descriptors. Fig. 3 shows a graphical display of how the shape descriptors for each patient behave over different levels of  $M$ . The perimeter, compactness and fractal dimension increase drastically with the increasing levels of  $M$ , as the

shape becomes more complex during the process. The area shows increases for lower  $M$  than slight decreases and at higher levels  $M$  increases. Bending energy and the descriptor  $\phi_l$  show drastically decreases and at higher levels monotone decreases. The eccentricity shows drastically decreases at lower levels  $M$  and monotone increases at higher levels. The numerical evaluation presented in Table 1a) and b) shows that in terms of absolute values, perimeter, compactness, fractal dimension are higher and the area is lower for patient A with severe deformations of the white brain matter (shape more complex) than for patient B with only subtle deformations.

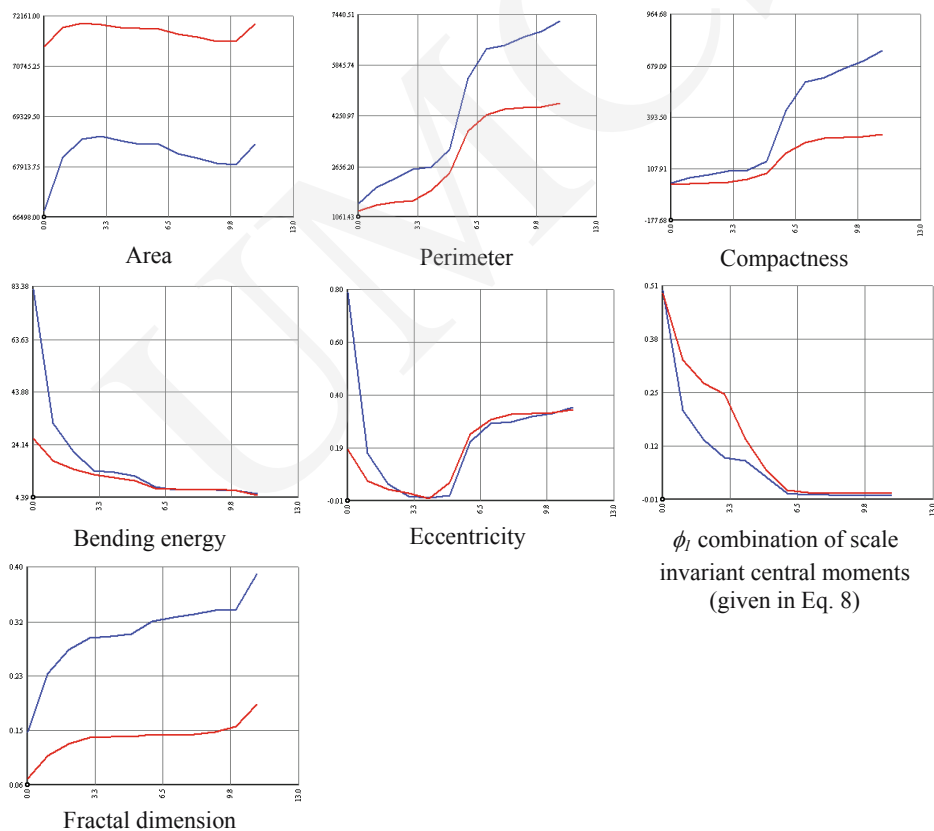


Fig. 3. Shape descriptions for reconstructed boundaries of different levels of  $M$  (frequency component given in Eq. 4) of patient A and patient B

## 6. Conclusions

In this paper shape representation and description techniques have been reviewed. Shape representation and description techniques can be classified into classes based on information extracted from the whole object or only from the

boundary: contour-based and region-based. In each class the techniques can be divided into structural and global. Also shape representation and description techniques can be classified into three categories based on the access of the shape properties: local, global and medial.

In order to describe and classify brain structures and examine shape description techniques the boundaries of white brain matters in one of the patients with severe deformations and in one of the patients with only subtle deformation were reconstructed using the Fourier descriptors for various frequency components. We evaluated the resulting reconstructed boundaries with the shape descriptors: region size, perimeter, compactness, bending energy, eccentricity,  $\phi_l$  (combination of scale invariant central moments) and fractal dimension. Our investigations show that shape descriptions over the object boundary become more complex (at higher frequency component). Our application for obtaining shape descriptors can be useful for clinicians to detect and analyze of the shape changes of the brain.

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